Sets, Permutations and Combinations

Conventionally, sets are denoted using curly brackets:

{Monday, Tuesday, Wednesday, Thursday, Friday}

{0}

{red, yellow, blue}

<u>very</u> small finite sets can be defined by listing their elements

later: sets are normally defined by boolean expressions!

The Membership Relation

$x \in S$ means x is a *member* of (or *element* of) set S.

$\in$ denotes the *membership relation*.

Eg. $a \in \{a,b\}$ $b \in \{a,b\}$.

$\mathbb{N}$ denotes the natural numbers

$0 \in \mathbb{N}$, $1 \in \mathbb{N}$

$S = T$ whenever they have the same elements.

e.g. $\{a,b\} = \{b,a\}$.

The number of elements in a set S is denoted by $|S|$.

cardinality of S.
pronounced mod S.

E.g.

$|\{\text{Monday, Tuesday, Wednesday, Thursday, Friday}\}| = 5$

$|\{0\}| = 1$

$|\{\text{red, yellow, blue}\}| = 3$

Cartesian Product

The *cartesian product* of sets S and T is denoted $S \times T$.
The elements of $S \times T$ are ordered pairs (s,t) where $s \in S$ and $t \in T$.

E.g. $\{0,1\} \times \{a,b,c\} = \{(0,a), (0,b), (0,c), (1,a), (1,b), (1,c)\}$

$|S \times T| = |S| \times |T|$

cartesian product of sets

multiplication of numbers

Note: $S \times T \neq T \times S$

$\{a,b,c\} \times \{0,1\} = \{(a,0), (b,0), (c,0), (a,1), (b,1), (c,1)\}$

Permutations

A *permutation* of a set is a way of arranging the elements of the set in order.

Eg. $\{a,b\}$ has two permutations: $a\,b$ and $b\,a$.

$2 = 2\times 1$

$\{a,b,c\}$ has six permutations: $6 = 3\times 2\times 1$

a b c	b a c	c a b
a c b	b c a	c b a

$\{a\}$ has one permutations. $1 = 1$

A set of cardinality n has $n\times(n-1)\times(n-2)\times\ldots\times 1$.
denoted $n!$ (pronounced n factorial).

$S!$ denotes the set of permutations of S. $|S!| = |S|!$.

$\subseteq$ is reflexive $[S\subseteq S]$.

$\subseteq$ is antisymmetric

$$[S = T \;\equiv\; S\subseteq T \wedge T\subseteq S].$$

$\subseteq$ is transitive

i.e. if $S\subseteq T$ and $T\subseteq U$ then $S\subseteq U$.

2^S denotes the set of subsets of S.

$|2^S| = 2^{|S|}$

(Other notations: $\mathcal{P}(S)$ $\mathbb{P}(S)$ "power set of S")

Subsets

Set S is a subset of set T, written $S \subseteq T$, if every element of S is an element of T.

E.g.

$$\{a\} \subseteq \{a, b\}$$

$$\{\text{red}, \text{green}\} \subseteq \{\text{red}, \text{yellow}, \text{green}, \text{blue}\}$$

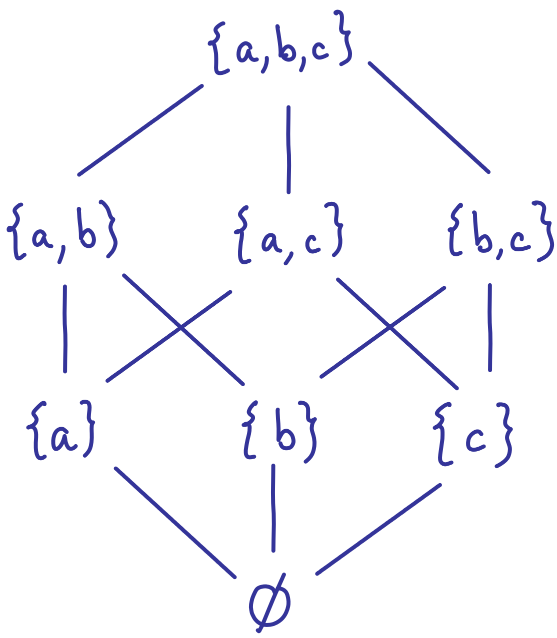
$$\{\text{magenta}\} \not\subseteq \{\text{red}, \text{green}\}$$

$\emptyset$ denotes the empty set. $\quad |\emptyset| = 0 \qquad \emptyset \quad 0$

$$[\, \emptyset \subseteq S \,]$$

$\subseteq$ is a partial ordering

Hasse Diagram of Subset Relation on $2^{\{a,b,c\}}$



$$\left|2^{\{a,b,c\}}\right| = 2^{|\{a,b,c\}|} = 2^3 = 8$$

Combinations

nCr is the number of subsets of cardinality r of a set of cardinality n.

— the number of ways of choosing r elements from a set with n elements.

Permutations

nPr is the number of ways of choosing r elements from a set of n elements *in order*. (I.e. choosing the 1st, the 2nd, ..., the rth.)

6 people are at the left bank of a river.
They want to get 3 of them across to the right bank.
Their boat will only carry 2 of them at a time.
How many different ways can this be done?